Whole Numbers

Topics Covered-

1. Introduction to Predecessor and successor

2. Whole Numbers

3. The Number Line

-Addition on the number line

- Subtraction on the number line

- Multiplication on the number line

4. Properties of Whole Numbers

- Closure property

- Commutative **of addition and multiplication**

- Associative of addition and multiplication

- Distributive of multiplication over addition

5. Patterns in Whole Numbers

- Patterns Observation

Introduction of Whole Numbers

Whole numbers are a set of numbers that include all the positive integers (numbers greater than zero) along with zero. They do not include any fractions or decimals.

Example - 0, 1, 2, 3, 4, 5…….

Predecessor of a Number

The predecessor of a number is simply the number that comes right before it in the sequence of natural numbers (whole numbers starting from 1). In simpler terms, if you have a number and you want to find its predecessor, you just need to subtract 1 from that number.

For example:

Let's say we want to find the predecessor of the number 6.

The predecessor of 6 is

$6-1=5$

Therefore, the predecessor of 6 is 5.

So, for any number n, the predecessor of n is

$n-1$

Let's try a few more examples:

1. Predecessor of 10:

$10-1=9$

Therefore, the predecessor of 10 is 9.

2. Predecessor of 1:

$1-1=0$

Therefore, the predecessor of 1 is 0.

Successor of a Number

The successor of a number is the number that comes right after it in the sequence of natural numbers (whole numbers starting from 1). In simpler terms, to find the successor of a number, you just need to add 1 to that number.

For example:

Let's say we want to find the successor of the number 6.

The successor of 6 is

$6+1=7$.

Therefore, the successor of 6 is 7.

So, for any number n, the successor of n is $n+1$.

Let's try a few more examples:

1. Successor of 10:

$10+1=11$

Therefore, the successor of 10 is 11.

2. Successor of 1:

$1+1=2$

Therefore, the successor of 1 is 2.

Operations on Number Line

The Number Line

A number line is a visual representation used to illustrate the order and relative values of numbers. It's a straight line on which numbers are marked at equal intervals. The number line extends infinitely in both directions, allowing us to represent any whole number, fraction, or decimal.

Here are some key points about the number line:

1. Basic Structure: The number line is a horizontal line with a starting point (usually representing zero) in the center. Numbers increase as you move to the right and decrease as you move to the left.

2. Marking Numbers: The number line is marked with evenly spaced points or tick marks, typically representing integers (whole numbers) at regular intervals. For example, you might see the number line marked with numbers like -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, and so on.

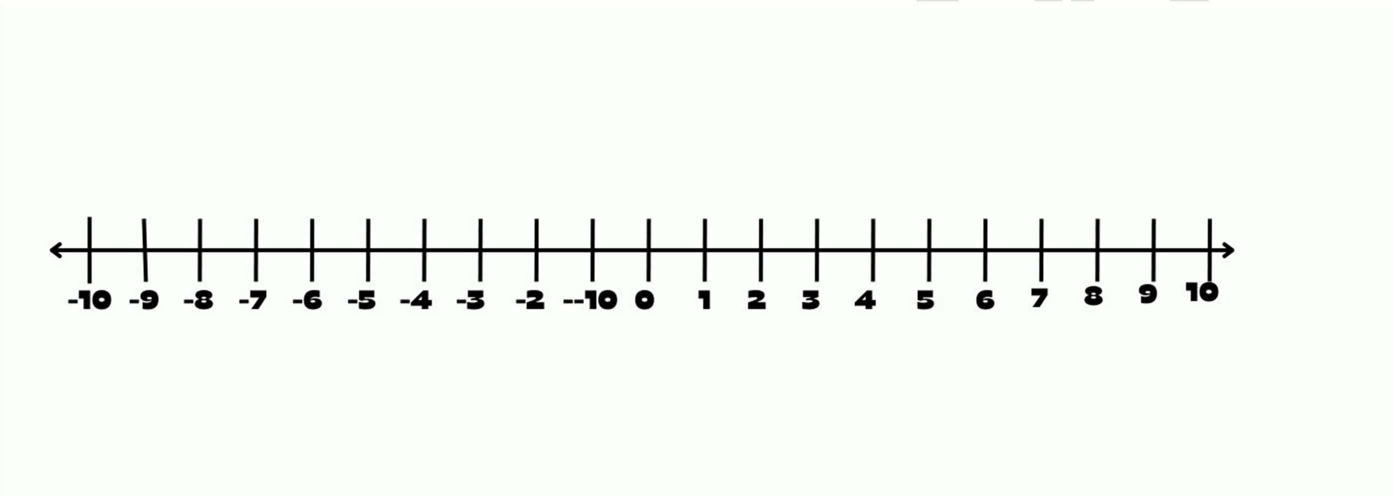
3. Position of Numbers: Each point on the number line corresponds to a specific number. For instance, the point directly to the right of 0 might represent 1, the next point represents 2, and so on. Similarly, moving to the left of 0 represents negative numbers (-1, -2, etc.).

4. Representing Intervals: The distance between consecutive integers on the number line is equal. For example, the distance between 0 and 1 is the same as the distance between 1 and 2.

5. Understanding Order: Students can use the number line to understand the concept of greater than (>) and less than (<). For instance, they can see that 3 is to the right of 2 on the number line, indicating that 3 is greater than 2.

6. Adding and Subtracting: The number line can be used to visualize addition and subtraction. To add a number, students move to the right by that number of units. To subtract, they move to the left.

7. Concept of Zero: The number line helps in understanding the concept of zero as the reference point. Positive numbers are to the right of zero, and negative numbers are to the left.



Addition on Number line

Addition is moving towards right on the number line.

Example: Let's add 3 + 2 using a number line.

1. Draw the Number Line: Draw a horizontal number line with evenly spaced marks or intervals. Label the key points, including zero (0) and the numbers you will be working with (in this case, from 0 to 5).

-5 -4 -3 -2 -1 0 1 2 3 4 5

2. Locate the First Number (3): Start at zero (0) on the number line and locate the first number, which is 3. Move 3 units to the right of zero.

-5 -4 -3 -2 -1 0 1 2 3 4 5

•

3

3. Add the Second Number (2): From the position of 3, count 2 more units to the right because we want to add 2 to 3.

-5 -4 -3 -2 -1 0 1 2 3 4 5

●

5

4. Identify the Sum: The final position on the number line (5) represents the sum of 3 + 2, which is 5.

Therefore

3+2=5.

In this example:

We started at 0.

We moved 3 units to the right to reach 3.

Then, we moved 2 more units to the right from 3 to reach 5, which is the sum of 3 + 2.

Subtraction on Number line

Subtraction is moving towards left on the number line.

Example: Let's subtract 5 - 2 using a number line.

1. Draw the Number Line: Draw a horizontal number line with evenly spaced marks or intervals. Label the key points, including zero (0) and the numbers you will be working with (in this case, from 0 to 5).

0 1 2 3 4 5 6

2. Locate the Starting Number (5): Start at zero (0) on the number line and locate the first number, which is 5. Move 5 units to the right of zero.

0 1 2 3 4 5 6

●

5

3. Subtract the Second Number (2): From the position of 5, count 2 units to the left because we want to subtract 2 from 5.

0 1 2 3 4 5 6

●

3

4. Identify the Difference: The final position on the number line (3) represents the result of 5 - 2, which is 3.

Therefore,

5−2=3.

In this example:

We started at 0.

We moved 5 units to the right to reach 5.

Then, we moved 2 units to the left from 5 to reach 3, which is the result of 5 - 2.

Multiplication on Number line

Multiplication is making jumps of equal sizes, several times on the number line, starting from zero.

Example: Let's multiply 4 × 3 using a number line.

1. Draw the Number Line: Draw a horizontal number line with evenly spaced marks or intervals. Label the key points, including zero (0) and the numbers you will be working with (in this case, from 0 to 12).

0 1 2 3 4 5 6 7 8 9 10 11 12

2. Locate the First Number (4): Start at zero (0) on the number line and locate the first number, which is 4. Move 4 units to the right of zero.

0 1 2 3 4 5 6 7 8 9 10 11 12

●●●●

4

3. Perform Repeated Jumps (Groups): From the position of 4, make three jumps of 4 units each (since we are multiplying by 3).

a) First Jump: Move 4 units to the right.

b) Second Jump: Move another 4 units to the right.

c) Third Jump: Move another 4 units to the right.

0 1 2 3 4 5 6 7 8 9 10 11 12

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4 8 12

Identify the Product: The final position on the number line (12) represents the product of 4 × 3, which is 12.

Therefore, 4×3=12.

In this example: We started at 0.We made three jumps of 4 units each (representing the multiplication by 3).

The final position reached on the number line (12) gives us the product of 4 × 3.

Properties of Whole Numbers

1. Closure Property of Whole Numbers

The Closure Property states that when we perform these operations on two whole numbers, the result is always another whole number.

Let's look at each operation and how the Closure Property applies to whole numbers:

1. Closure Property of Addition:

When you add two whole numbers together, the result is always a whole number.

For example:

2+3=5 (Both 2 and 3 are whole numbers, and their sum 5 is also a whole number.)

7+4=11 (Both 7 and 4 are whole numbers, and their sum 11 is also a whole number.)

Therefore, addition of whole numbers is closed under addition.

2. Closure Property of Subtraction:

When you subtract one whole number from another, the result is not always guaranteed to be a whole number. However, if the result is a whole number, it still satisfies the Closure Property.

For example:

5−3=2 (Both 5 and 3 are whole numbers, and their difference 2 is also a whole number.)

8−6=2 (Both 8 and 6 are whole numbers, and their difference 2 is also a whole number.)

Therefore, subtraction of whole numbers is generally not closed under subtraction, but the resulting whole number does satisfy closure.

3. Closure Property of Multiplication:

When you multiply two whole numbers together, the result is always a whole number.

For example:

4×3=12 (Both 4 and 3 are whole numbers, and their product 12 is also a whole number.

6×2=12 (Both 6 and 2 are whole numbers, and their product 12 is also a whole number.)

Therefore, multiplication of whole numbers is closed under multiplication.

4. Closure Property of Division:

When you divide one whole number by another (where the divisor is not zero), the result is not always guaranteed to be a whole number.

However, if the division results in a whole number, it satisfies the Closure Property.

For example:

8÷2=4 (Both 8 and 2 are whole numbers, and their quotient 4 is also a whole number.)

10÷5=2 (Both 10 and 5 are whole numbers, and their quotient 2 is also a whole number.)

Therefore, division of whole numbers is generally not closed under division, but the resulting whole number does satisfy closure.

Commutative Property of Whole Numbers

This property states that changing the order of the numbers being added or multiplied does not change the result.

Let's look at the Commutative Property in the context of addition and multiplication for whole numbers:

1.Commutative Property of Addition:

The Commutative Property of addition states that the order in which we add two whole numbers does not affect the sum.

In mathematical terms:

a+b=b+a for any two whole numbers

a and b

For example:

2+3=3+2 (Both sides result in 5.)

7+4=4+7 (Both sides result in 11)

Therefore, the order of numbers in addition can be changed without changing the sum.

Commutative Property of Multiplication:

The Commutative Property of multiplication states that the order in which we multiply two whole numbers does not affect the product.

In mathematical terms:

a×b=b×a for any two whole numbers a and b.

For example:

4×3=3×4 (Both sides result in 12).

6×2 = 2×6 (Both sides result in 12.)

Therefore, the order of numbers in multiplication can be changed without changing the product.

Key Points:

a) The Commutative Property holds true only for addition and multiplication, not for subtraction or division.

b) This property allows us to rearrange the numbers involved in addition or multiplication without affecting the final result.

c) Understanding the Commutative Property helps simplify calculations and solve problems more efficiently.

Associative property of Numbers

This property states that the grouping of numbers being added or multiplied does not affect the result.

Let's explore the Associative Property in the context of addition and multiplication for whole numbers:

1. Associative Property of Addition:

The Associative Property of addition states that when adding three or more whole numbers, the way we group them does not affect the sum.

In mathematical terms:

$(a+b)+c=a+(b+c)$ for any three whole numbers a, b and c

For example: $(2+3)+4=2+(3+4)$ (Both sides result in 9).

$(7+4)+2=7+(4+2)$ (Both sides result in 13).

Therefore, the way we group numbers in addition (associating left or right) does not change the sum.

2.Associative Property of Multiplication:

The Associative Property of multiplication states that when multiplying three or more whole numbers, the way we group them does not affect the product.

In mathematical terms:

$(a\times b)\times c=a\times(b\times c)$ for any three whole numbers a, b and c

For example:

$(4\times3)\times2=4 \times (3\times2)$ (Both sides result in 24).

$(6\times2)\times3=6 \times (2\times3)$ (Both sides result in 36.)

Therefore, the way we group numbers in multiplication (associating left or right) does not change the product.

Key Points:

a) The Associative Property holds true only for addition and multiplication, not for subtraction or division.

b) This property allows us to regroup the numbers involved in addition or multiplication without affecting the final result.

c) Understanding the Associative Property helps simplify complex calculations and solve problems more efficiently.

It's important to note that this property specifically applies to whole numbers in the context of addition and multiplication.

Distributive property of Numbers

It shows how multiplication can be distributed over addition or subtraction. This property is often introduced in elementary school around the 6th grade level.

The distributive property states that for any numbers a, b and c

a×(b+c)=a×b+a×c

In other words, when you have a number a multiplied by the sum of

b and c, you can distribute a to each term inside the parentheses (the addition), resulting in the product of a and b added to the product of a and c.

Here's a simple example to illustrate the distributive property:

Example:

Let's take

a=3,

b=4, and c=2.

We want to evaluate

3×(4+2).

Using the Distributive Property:

3×(4+2)=3×4+3×2

Calculate the Multiplications:

3×(4+2)=12+6

Add the Results:12+6=18

Therefore, 3×(4+2)=18

Here, we distributed 3 across the addition inside the parentheses:

3×(4+2)=3×4+3×2=12+6=18

This example shows how the distributive property allows us to break down a multiplication involving addition into simpler steps.

Identity Numbers

Identity numbers refer to special numbers that have unique properties when combined with certain operations. The two main identity numbers students encounter at this level are the identity element for addition (also known as the additive identity) and the identity element for multiplication (also known as the multiplicative identity).

1. Additive Identity:

The additive identity is the number that, when added to any other number, leaves the other number unchanged. For real numbers, the additive identity is 0. This means that for any number a:

a+0=a

For example:

5+0=5

−3+0=−3

So, 0 is the additive identity because adding 0 to any number a does not change the value of a.

2. Multiplicative Identity:

The multiplicative identity is the number that, when multiplied by any other number, leaves the other number unchanged. For real numbers, the multiplicative identity is

1

1. This means that for any number a:

$a \times 1 = a$

For example:

$6 \times 1 = 6$

$(-2) \times 1 = -2$

So, 1 is the multiplicative identity because multiplying any number a by 1 does not change the value of a.

In summary:

Additive Identity: 0 because $a+0=a$ for any number a.

Multiplicative Identity: 1 because $a \times 1 = a$ for any number a.

Patterns Using Whole Numbers

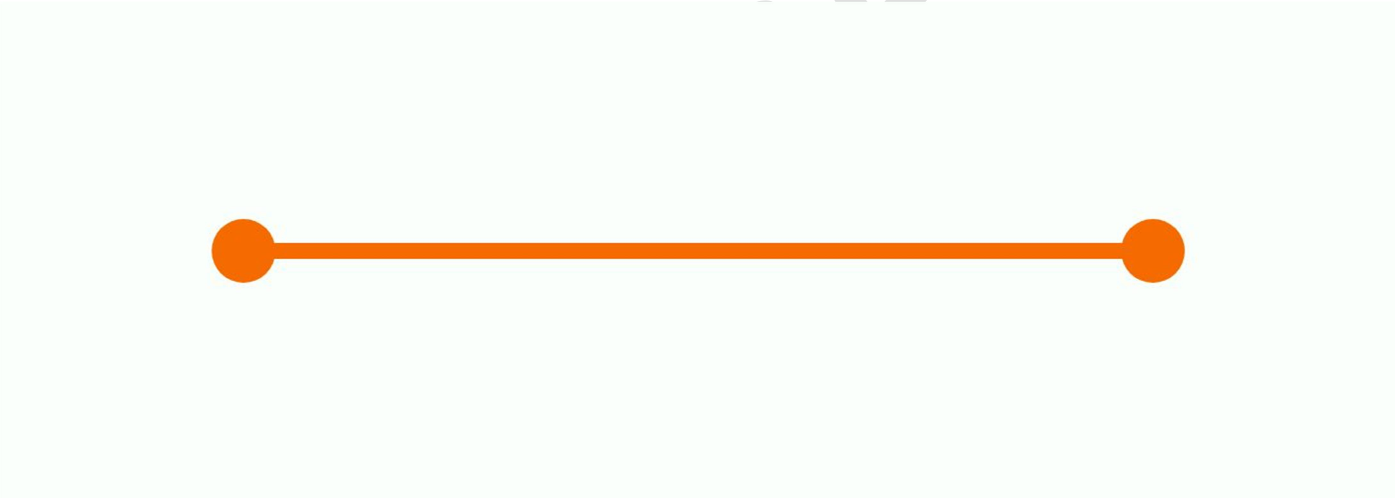
We can use whole numbers to make different patterns like straight line, triangle, square, and rectangle.

Let's suppose one point represents the number 1 and two points represent the number 2, and so on. We can create different patterns with a given set of points.

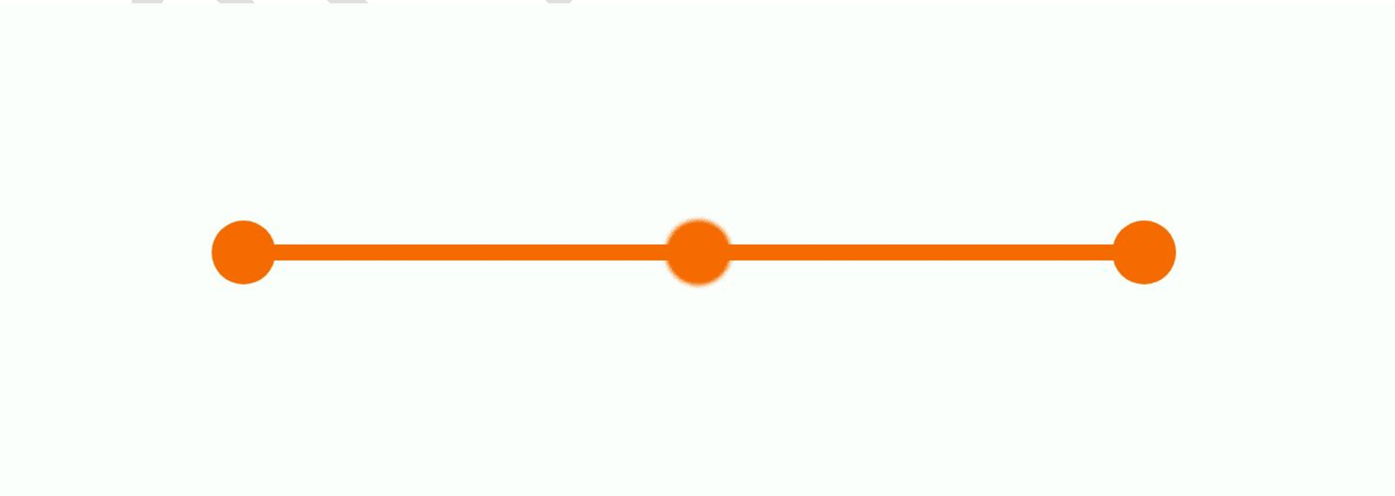
With 1 point, we cannot create any patterns. It always remains a point.

1. Straight Line

If two points are given, then the two can be joined together in the shape of a straight line.



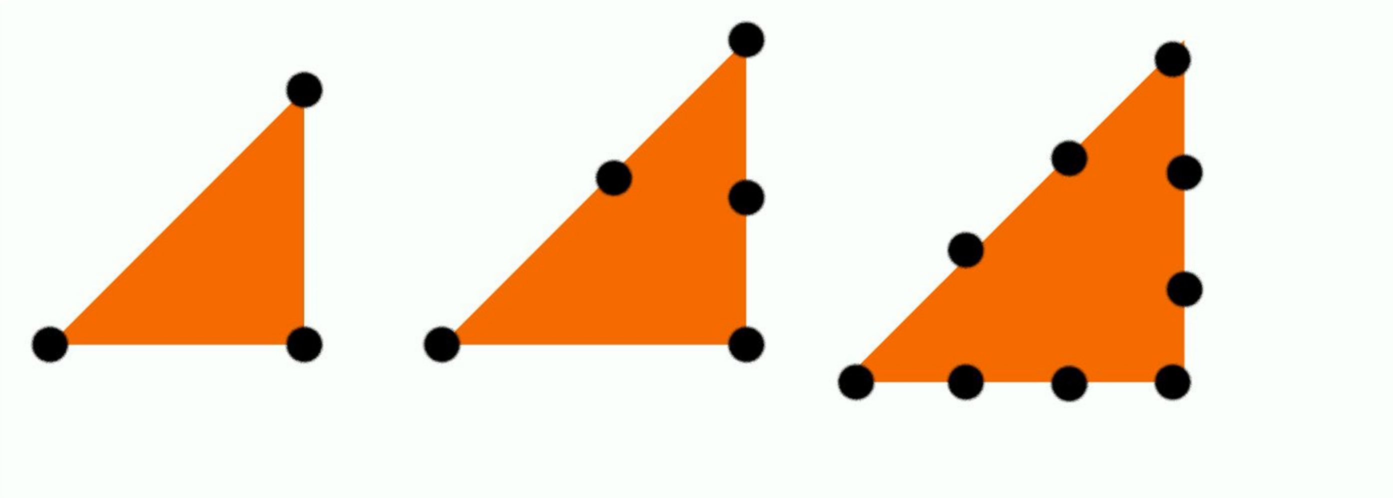
With one more point, we can still form a straight line.



We can form a straight line with any number of points, except 1.

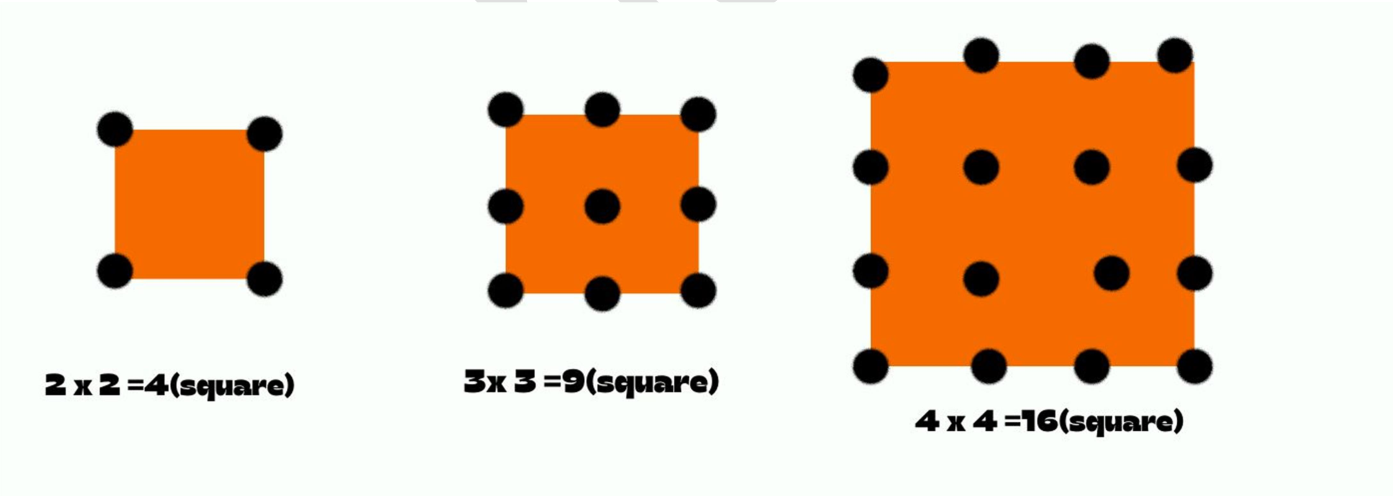
2. Triangular numbers

Numbers that form a triangle are called triangular numbers. For example, 3, 6, 10, 15, and 21 are triangular numbers.



Square Numbers

We get a square number by multiplying a whole number with itself. 4, 9, 16... are the numbers that form square patterns.



If we add any two consecutive triangular numbers, we end up getting a square number. For example,

3 + 6 = 9